### Applied analysis of partial differential equations inverse problems

### Task 1. Transformation inverse problems to optimization problems

Consider heat transfer phenomenon. The mathematical model of the system is the heat equation with boundary conditions



where *u* is the temperature, *t*  is the time, *x* is the spatial variable, *L* is the length of the body, *ρ* is the density, *c* is the heat capacity, *λ* is the thermal conductivity, *f* is the heat source, *ϕ* is the initial temperature, *a* is the boundary temperature of the left end of the body, and *b* is the boundary temperature of the right end.

**Variants**

|  |  |  |
| --- | --- | --- |
| variant | unknown parameter | measurable information |
| 1 | *f, ρ* | ; |
| 2 | *a, ϕ* | ; |
| 3 | *λ, b* | ; |
| 4 | *c*, *L* | ; |
| 5 | *ρ*, *ϕ* | ; |
| 6 | *c*, *a* | ; |
| 7 | *f, ϕ* | ; |
| 8 | *b*, *λ* |  |
| 9 | *f, L* |  |
| 10 | *ϕ*, *b* |  |
| 11 | *ρ*, *a* |  |
| 12 | *c*, *b* |  |
| 13 | *a, L* | ; |

**The inverse problem**: find the unknown parameters such that the solution *u=u*(*x*,*t*)of the system satisfies the given additional conditions

**It is necessary** to transform the given inverse problem to the corresponding optimization problems.